Invited Session

All Roads Lead to Lisbon: Some Modeling Approaches for Designing Optimal Networks

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Abstract

The Diameter-Constrained Minimum Spanning and Steiner Tree Problems seek a least cost spanning or Steiner tree subject to a (diameter) bound imposed upon the number of edges in the tree between any node pair. Using a traditional multicommodity flow model with a commodity for every pair of nodes, we were unable to solve a 20-node and 100-edge spanning tree problem after one week of computation, whereas with new models, we could solve this specific problem in less than one second and larger problem instances with up to 100 nodes and 1000 edges. The largest model contains more than 250,000 integer variables and more than 125,000 constraints. The improved models conceive of diameter-constrained trees as being composed of several core subproblems, including directed trees and hop-constrained paths. Situations with odd diameters prove to be more difficulty to solve; we show that apparently different modeling approaches (models with intersecting trees, those with "shortest" and "longest" paths, and a disjunctive modeling approach) for these situations all lead to equivalent linear programming relaxations of the problem. We conclude with some thoughts about lessons learned for solving network design problems more generally.